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Three Approaches to Improve the Estimation Accuracies of the Vertical VLBI Station Positions

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VLBI 局の鉛直位置推定精度を上げるための三つの方法

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要　旨

測地 VLBI において，局の水平面内の位置を正確に求めるのは比較的簡単だが，鉛直方向の位置を精度よく決定するためには特別な注意が必要である．この注意は具体的に次の三つの点に集約できる．第一は実験のスケジューリングにおける注意であり．時刻オフセット及び大気遅延と局鉛直位置の相関を軽減するために低い仰角の観測を積極的に取り入れることである．第二はデータ解析における注意で，大気遅延の時間変化を正確に反映させることが残差の軽減に重要である．そのためには実験時間を数多くの区間に分けて大気を推定する必要があるが，解の安定性を保つためには大気状態の時間的連続性をゆるく束縛することが効果的である．最後に鉛直位置の推定値は大気のマッピング関数によって変化するので，なるべく精度のよいモデルを選択することが系統誤差を除去する上で重要となる．

ABSTRACT

It has been pointed out that very long baseline interferometry (VLBI) is less accurate in determining the station positions in a vertical axis than in a horizontal plane. However, possible applications of VLBI for sea level monitoring, terrestrial reference frame for astrometry and earth rotation studies, etc., require accurate determination of the station position in the vertical axis as well as in the other axes. I propose three different approaches to improve the accuracies of the estimated vertical site positions, that is, to control the vertical elongation of the positional error ellipsoids by low elevation observations, to reduce the size of the ellipsoids by decreasing the residuals, and to select a "right" atmospheric mapping function to avoid systematic errors in vertical positions.

1. Introduction

Geometric delays measured in very long baseline interferometry (VLBI) are used to estimate various geodetic parameters, e.g., earth's rotational parameters, antenna positions in geocentric rectangular coordinates. Repeated baseline length measurements for plate motion and deformation studies are being organized as international VLBI experiments

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such as Crustal Dynamics Project (CDP). In these experiments, lateral station movements have been more emphasized than their radial movements mainly because we are more interested in the horizontal movement like plate motions and partly because VLBI is not capable of determining vertical positions accurately and less sensitive to the vertical station movements.

Recently, global sea level changes are receiving much interest coming from our concern over global environments. However, in order to remove apparent sea level changes in tidal stations due to vertical crustal movements, e.g., glacial rebound and local tectonic movements, these stations should be connected to nearby space geodetic stations whose vertical positions are tightly linked to other stations in the world. VLBI stations could also be constituents of "terrestrial reference frame" for earth rotation studies and astrometry. All of the positional components should be well determined for this purpose. In this paper, I will describe what is important to improve the accuracy of vertical station positions to make geodetic VLBI results useful in these studies.

2. Error ellipsoids of station positions

One geodetic VLBI observing session, or an "experiment", is composed of 100 to 200 simultaneous "observations" of various celestial radio sources by multiple stations. After cross correlation processing, we get 100 to 200 delay "data" (which should be multiplied by the number of baselines if stations are more than two). Various geodetic parameters (e.g., station positions, earth orientation) and non-geodetic parameters (e.g., clock offsets, thickness of neutral atmosphere) are "estimated" by the weighted least-squares method from these data.

Station positions are usuallly estimated in terms of a geocentric Cartesian coordinate but I discuss their errors in local north, east and up components which can be obtained as a simple coordinate rotation. The error of the estimated station position is expressed as an "ellipsoid". Their shapes are usually "prolate", i.e., elongated in a vertical direction because of the between-parameter correlations discussed in the next chapter. In Fig. 1 are illustrated three possible approaches to reduce the vertical errors. The first

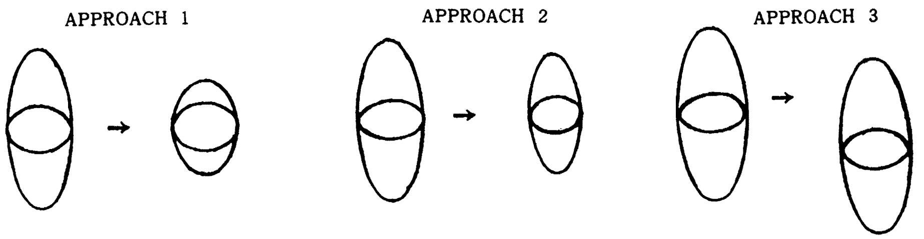

Fig. 1. Three approaches toward the better estimation of the vertical positions of VLBI stations. They are, from left to right, suppression of elongation, reduction of the size and removal of systematic errors.

one is to suppress the vertical elongation. The second one is to make the overall size of the ellipsoid small. These two are to reduce "random" errors. As the last one, I discuss the existence of "systematic" vertical errors which might be caused by atmospheric model differences.

3. Three approaches to improve vertical VLBI station position accuracies

3-1. How to reduce the elongation of the ellipsoid

We have to know "why" they have elongated shapes in order to discuss "how" we can suppress the vertical elongation. It is convenient to define two factors which determine the estimation error of a certain parameter. They are the "sensitivity" of the data with respect to the parameter and its "distinguishability" from other parameters. The first factor is evaluated as the sum of the squares of partial derivatives, i.e., the length of the column vector in the Jacobian matrix. The second one is, in other words, the orthogonality of parameter pairs, i.e., the smallness of the dot products between the column vectors.

Now I discuss how the errors in the three components of the station positions are influenced by these two factors. I assume the clock offsets and atmospheric zenith delays as additional non-geodetic parameters to be adjusted simultaneously (they actually vary with time but are assumed constant for the sake of simplicity). Stars are assumed to be distributed randomly in the sky and between-site mutual sky visibility is assumed to be perfect. The sensitivity, i.e., the absolute values of positional partial derivatives are the largest (zero) if the star direction is coincident with (perpendicular to) the coordinate axis. Hence, among the three components, there will be little difference in "sensitivity", that is, the square-sum of these derivatives as far as the star directions are selected randomly. The situation for the senses of the partial derivatives, however, are different. As for the partial derivatives with respect to the north-south axis, their senses could be alternated by observing the stars of northern and southern azimuths ("dual-sensed"). So could be those for the east-west axis. However, the partial derivatives for the up-down (vertical) axis are always of a single sense ("single-sensed") as we cannot observe radio sources in a negative (downward) elevation (Fig. 2a).

Partial derivatives with respect to the clock offset are constant throughout the experiment (Fig. 2b). Those with respect to the atmospheric thickness are approximated by the cosecant of the elevation angle (Fig. 2c). These two parameters are also "single-sensed" and this feature causes the degradation of the mutual "distinguishability" among the three parameters, i.e., the vertical site position, clock offset and atmospheric thickness. Fig. 3 compares the ellipsoidal shapes for various combinations of the parameters. If only the station position is unknown (Fig. 3A), the error "ellipsoid" does not have significant elongation. If we assume the complete set of parameters (site position plus clock, atmosphere Fig. 3D), the vertical elongation is the most remarkable. For the combination of the site position and either one of clock and atmosphere, the elongation is

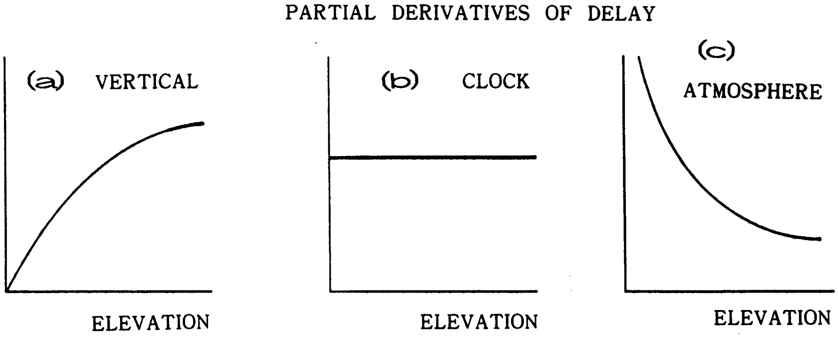

Fig. 2. Elevation dependence of the partial derivatives of the delay with respect to three parameters, i.e., vertical site position, clock offset and atmospheric thickness.

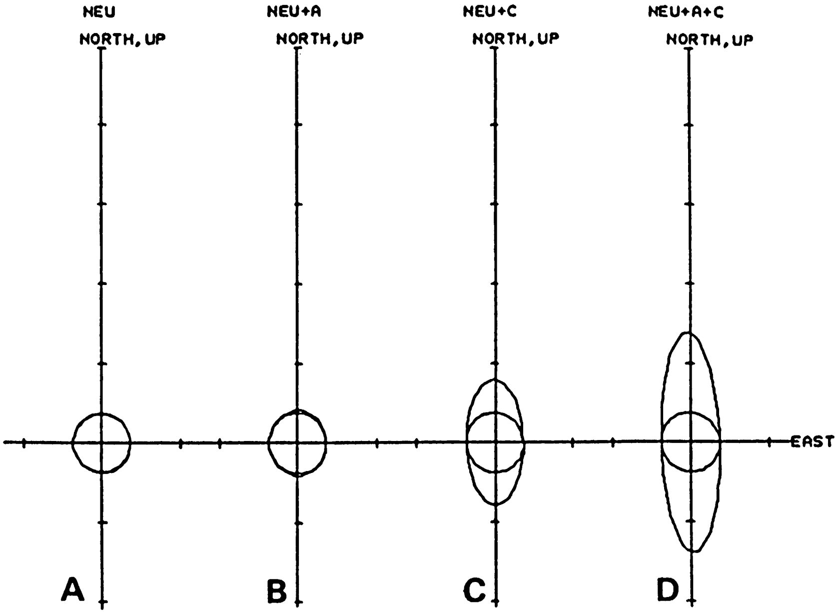

Fig. 3. Error ellipsoids of station positions for four different combinations of the parameters (from left to right, only station position, position+atmosphere, position+clock, position+atmosphere+clock). They are obtained by computer simulations under following assumptions; 200 observations in 24 hour experiment, 0.1 ns for delay observational errors, uniform distribution of stars. Ellipsoids are shown as those projected onto NS-EW and UD-EW planes. One division corresponds to 1 cm.

intermediate (Fig. 3B, C). These results demonstrate that the vertical elongation is due completely to the poor distinguishability of the vertical component from the clock and atmosphere and not due to the low sensitivity of the delay data to the vertical position changes.

The partial derivatives for these three parameters have different elevation dependences (Fig. 2) and this enables them (although not very well) to be separated from each other.

Hence, it is essentially important to make elevation range as wide as possible. Low elevation observations are especially important because the contrast between "vertical" (approaching zero) and "atmosphere" (approaching infinity) becomes higher as the elevation approaches to zero. The dependence of the ellipsoid shapes on the elevation ranges shown in Fig. 4 demonstrates that the degree of elongation is very sensitive to its lower limit (Fig. 4a) while high elevation observations are less important (Fig. 4b).

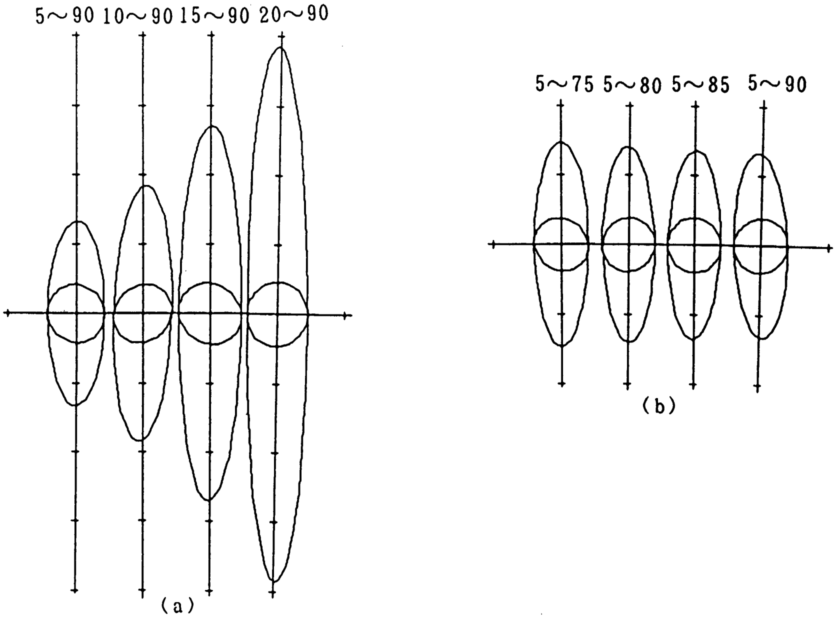

Fig. 4. Error ellipsoids of station positions for different elevation ranges. The conditions are same as Fig. 2 except star elevation restrictions.

Involving as many low elevation observations as possible in a schedule can be achieved by making it a rule to observe the stars just after the rising and before the sinking. It is also suggested that the site for a geodetic VLBI antenna should be selected from the places with small horizontal masks so that low elevation observations can be realized in a real schedule. Antennas for radio astronomical use are often constructed in basins surrounded by mountain ranges in order to avoid ground-wave radio interferences but such a place is not ideal for geodetic purposes.

In an actual VLBI experiment, it is sometimes difficult to keep azimuthal symmetry due to the restriction in the mutual sky visibility. If observation azimuths concentrate on a certain direction, positional error ellipses in a horizontal plane tend to elongate in this direction (Fig. 5) by the same reason as the vertical elongation discussed here. In a long and single baseline VLBI experiment, mutual visibility is heavily restricted and it is better to avoid such a case by having an additional station between them and by doing a lot of "sub-net" observations, i.e., the observations by only a certain portion of the whole network.

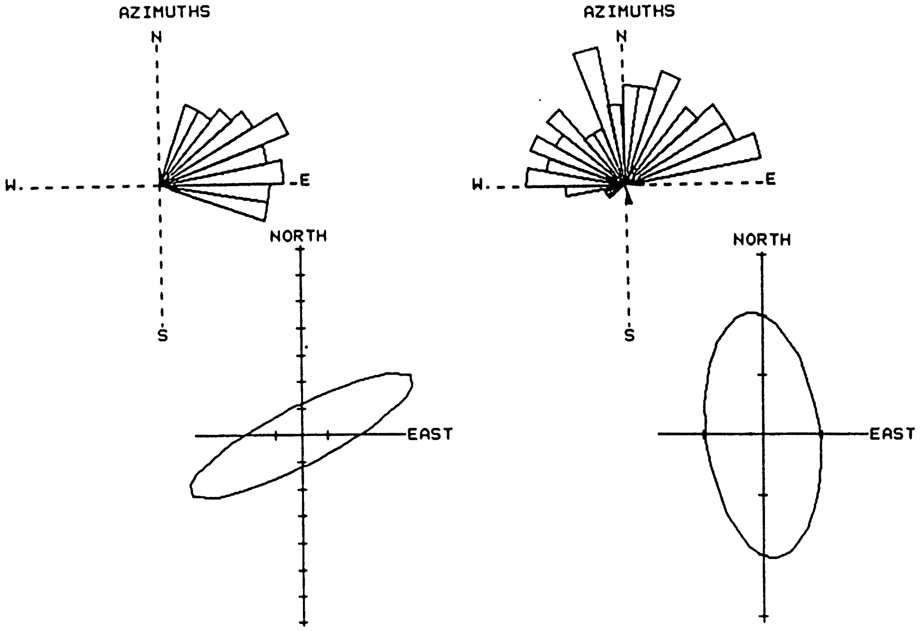

Fig. 5. Distribution of observational azimuths and the shape of error ellipses of the station positions in a horizontal plane. Azimuthal asymmetry causes elongation of the ellipses in the direction of the azimuth concentration.

3-2. How to reduce the overall size of the ellipsoid

An overall ellipsoidal size is determined by observational errors while its shape is controled by an observational schedule. Observed delays have their a-priori errors deduced from receiving band-widths and signal-to-noise ratios after cross correlation (KAWAGUCHI, 1983). The post-fit delay residuals should be almost as large as such observational errors, i.e., chi-squares divided by the degree of freedom should not seriously deviate from unity. However, in usual VLBI experiments, the actual residuals are much larger than such observational errors suggesting that some "unmodeled" physical effects causing systematic delay behaviors still remain.

We usually introduce "hypothetical" errors which correspond to such unmodeled effects and add them to a-priori observation errors. The amount of these hypothetical errors are determined empirically so that the chi-square over the degree of freedom becomes unity. Actually, such a-posteriori errors are much larger than a-priori errors, that is, the amounts of the hypothetical errors actually determine the final ellipsoidal sizes. I want to demonstrate that the considerable part of this unknown effect comes from the rapid temporal variation of the neutral atmosphere thickness.

An effective way to estimate values with temporal variations is to divide them into several "intervals" and represent the variation as a line with "breaks" at their boundaries. Residuals tend to get smaller as the interval become shorter. However, the number of

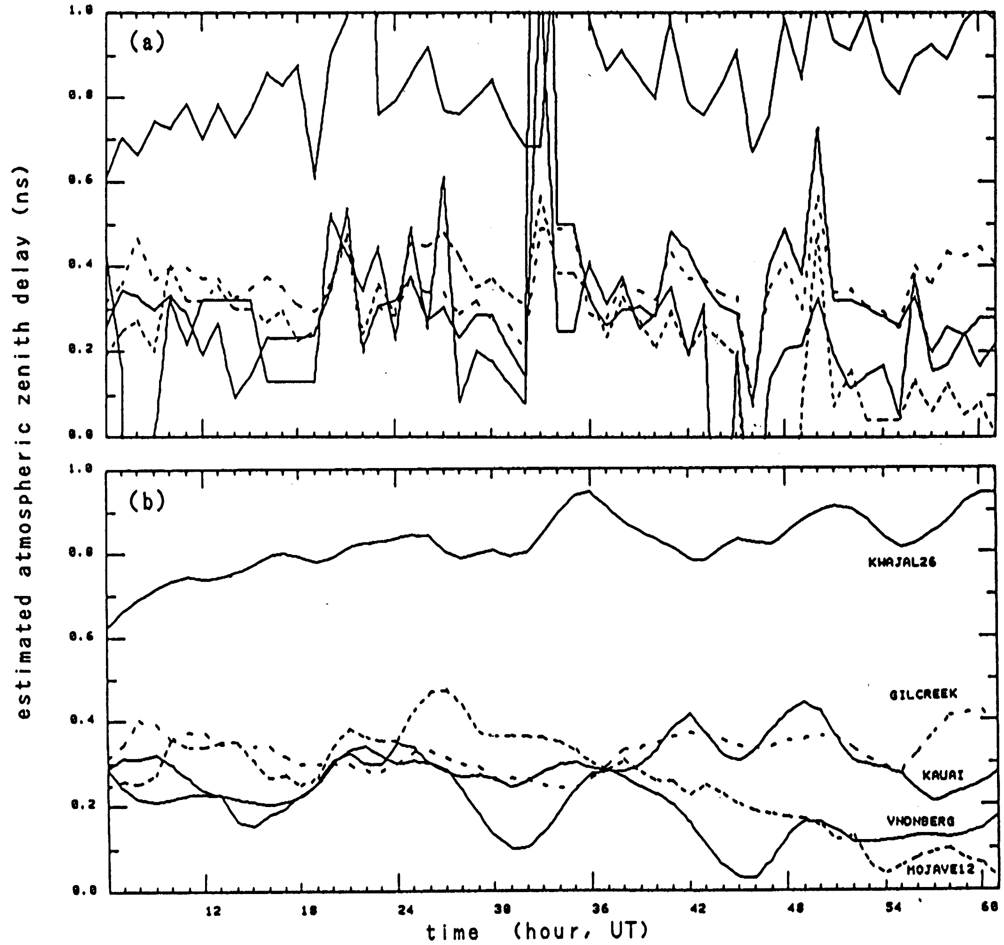

Fig. 6. Estimated atmospheric zenith delays (vertical axis) versus time (horizontal axis). Because the data are already corrected for the "dry" components, estimated value are those for "wet" components. They are estimated as lines with breaks at each one hour. No constraints for "a" and continuity constraints are introduced in "b" (tolerance is 50 ps/hr). The experiment is 1984 EPAC-1.

observations per interval becomes also smaller and this causes unstable behaviors of the estimated atmospheric values (Fig. 6a). We could stabilize estimated values by introducing weak "constraints" for the between-interval continuities. Such constraints are implemented in algorithm by adding another term to be minimized in the least-squares procedures, that is,

$$\sum_i (yo_i - yc_i)^2/\sigma_i^2 + \sum_j (x_j - x_{j-1})^2/t_j^2 \longrightarrow \text{minimum},$$

in which yo_i and yc_i are the i-th observed and calculated delays respectively and x_j is the estimated values of the j-th parameters. The second summation is done only over the adjacent pairs of atmospheric intervals. "t" is the "tolerance", i.e., the acceptable between-interval difference of the atmosphere. (There are two conceivable approaches to determine the most suitable value for the tolerances; one is to adopt the values based on a study of the actual meteorological observations, and the other is to adopt the value

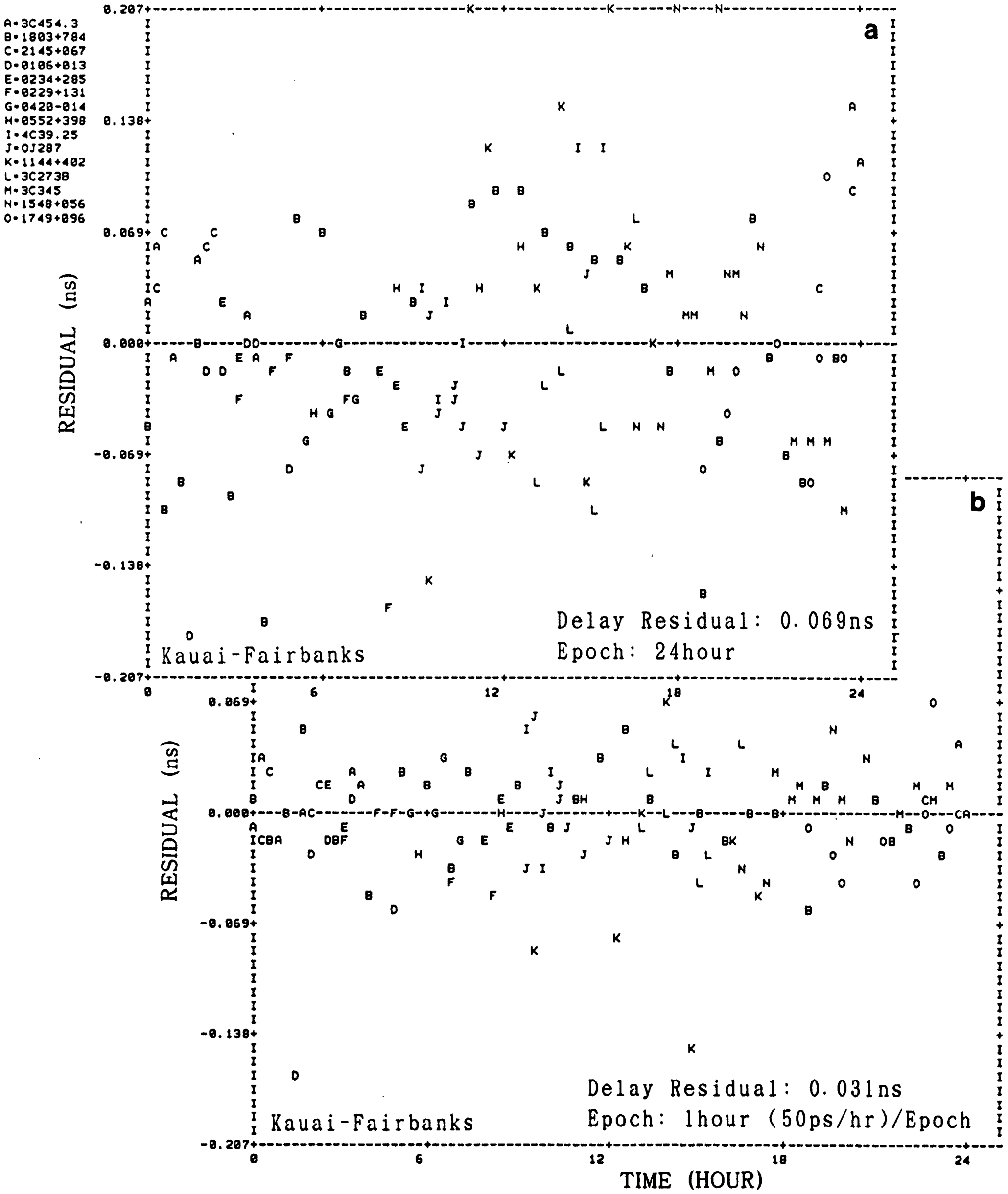

Fig. 7. Delay residuals (vertical axis) versus time (horizontal axis). 24 hour duration is not divided into intervals in "a" and is divided into 24 one-hour intervals in "b", where systematic behaviors of delay residuals disappear and residuals become significantly smaller. Experiment is 1988 XPAC-1 and the baseline is Kauai-Fairbanks. Different alphabets denote different radio sources listed to the left. Tolerance in "b" is same as Fig. 6.

which minimizes the scatter of the baseline lengths around the best-fit line. Although preliminary tests show that the adjustments/formal errors of site positions are not sensitive to the selection of tolerances, to optimize the constraint will be an important future problem.) We can see the effect of the constraint by comparing Fig. 6a and Fig. 6b. Fig. 6a shows the estimated atmospheric temporal variation without any constraints while in Fig. 6b the continuity constraints are on for the same data.

With this technique it becomes possible to make interval lengths sufficiently short, say, about one hour. Fig. 7 compares the delay residual between the analysis with only one interval (i.e., no breaks) for the atmosphere and the analysis where 1 day duration is divided into 24 intervals with the length of one hour. It is seen that systematic delay behaviors in the first analysis disappear in the second analysis and that residual delay weighted-root-mean-squares (WRMS) is less than a half in the second analysis. Decrease of delay residuals corresponds to the decrease of "hypothetical" additional delay observational errors and this results in the decrease of the overall size of the positional error ellipsoid. This technique is similar to the method which has already been routinely used in some geodetic VLBI data analysis group, sometimes referred to as "auto-constrained" method (MA *et al.*, 1989).

3-3. Systematic errors in vertical positions

As long as the deviation of the estimated vertical position from the true value is "random", we can reduce the error by averaging sufficient number of data. However, if the errors are "systematic", such an average still remains to be a biased estimation.

What gives rise to the systematic errors of the vertical station positions is the "wrong model", namely wrong partial derivatives of the delay with respect to the vertical position and to other parameters which have significant correlation with the vertical, e.g., clock offsets and atmospheres. Those with respect to the vertical position (sine of the elevation angle) and to the clock (constant) are too simple to be wrong (Fig. 2). However, the partial derivative with respect to the atmospheric zenith delay, i.e., the mapping function (a function to express the ratio between zenith delay and the line-of-sight delay), is less well established.

The basic form of mapping functions is the cosecant of the elevation angle but the formulas actually used in VLBI data analyses are relatively complicated since the earth surface curvature and the atmospheric refraction should be taken into account to obtain centimeter accuracies of excess path lengths. It is possible to know if a mapping function is correct by the "elevation cut-off test", the test to compare the estimated station positions between the solution with entire data set and that with a subset consisting of the data whose elevations are higher than a certain cut-off value. DAVIS *et al.* (1985) concluded that conventional mapping functions such as those by CHAO (1974) and MARINI and MURRAY (1974) are not correct in low elevations and proposed a new mapping function called CFA 2.2. This and another model proposed by LANYI (1984) are currently used as standard atmospheric mapping functions for geodetic VLBI.

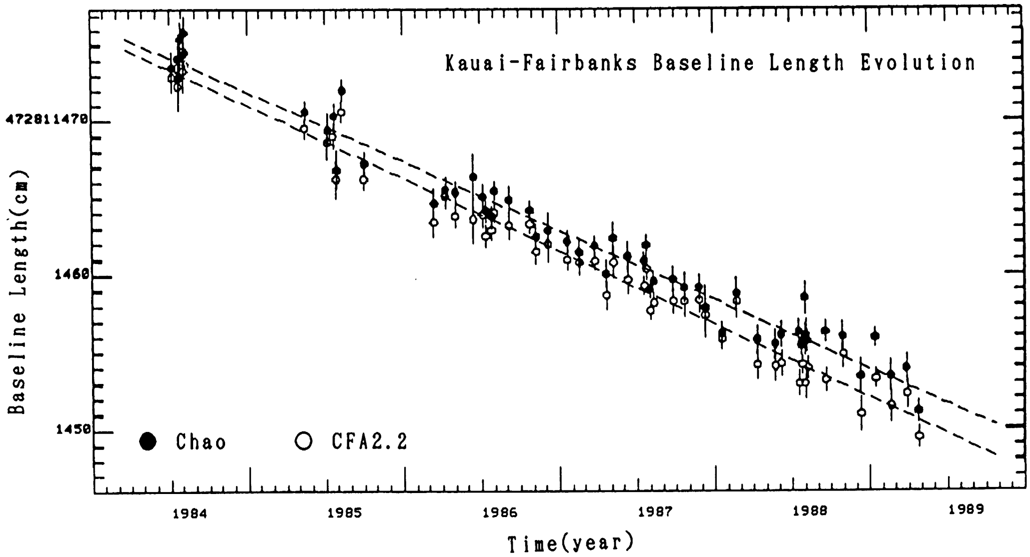

Fig. 8. Kauai-Fairbanks baseline estimated lengths from 1984 to 1989 obtained using Chao (solid circles) and CFA 2.2 (open circles) atmospheric mapping functions. Broken lines are the best-fit lines for these two sets.

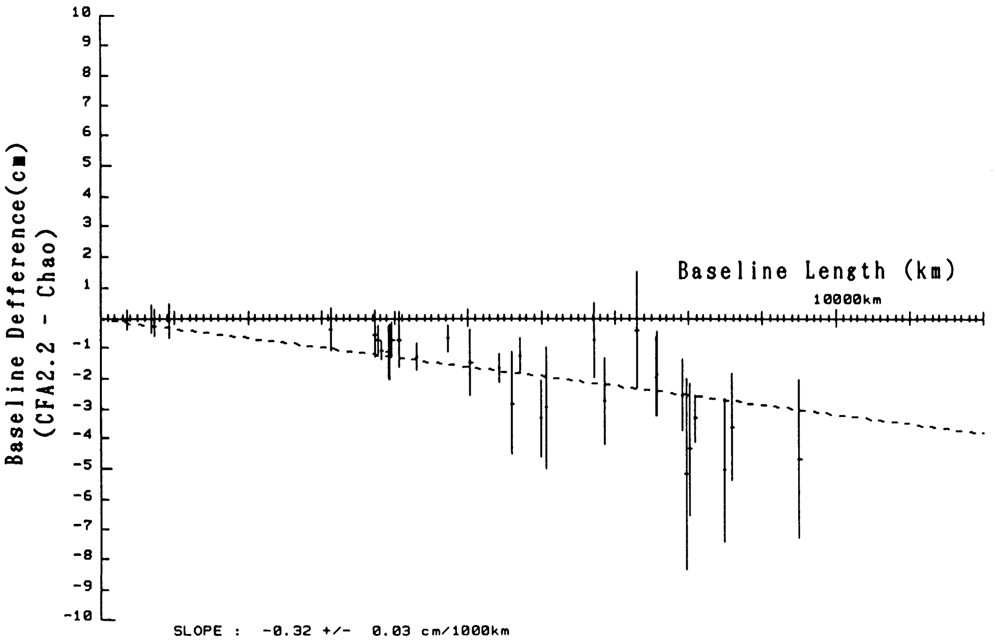

Fig. 9. Comparison between baseline lengths (horizontal axis) and the differences of the estimated baseline lengths (vertical axis) between Chao and CFA 2.2. Their proportional relationship indicates that these differences originate from vertical station position differences. Broken line is the best-fit line whose slope corresponds to about 3 mm difference per 1000 km baseline.

Here I want to take Chao and CFA 2.2 functions and evaluate the systematic differences between the estimated vertical positions with these functions. The most important difference between them is that the coefficients appearing in CFA 2.2 are the functions of five meteorological parameters, that is, surface temperatures/pressures/humidities and tropopause heights/temperatures while those for Chao function are fixed. Actually, it is most strongly dependent on the surface temperature and less dependent on the tropospheric height/temperature and hardly sensitive to the other parameters such as surface pressure/humidity.

Fig. 8 compares two sets of the baseline lengths estimated using Chao and CFA 2.2 functions. They run parallel to each other with a slight but significant systematic "offsets" possibly due to systematic difference in the estimated vertical site positions. By plotting the offsets of various baselines as a function of baseline lengths, we can confirm that the vertical position differences are really responsible for these offsets. The direction cosine between the local vertical and the baseline vector is proportional to the baseline lengths. Accordingly, the length offsets due to vertical position differences should be proportional to the baseline lengths. Fig. 9 shows the relationship between them. In this figure, their linear relationship is clear and this indicates that such offsets are due to the systematic errors in estimated vertical positions caused by the difference of atmospheric mapping functions.

4. Conclusion

In order to estimate the vertical components of VLBI station positions accurately, what we have to keep in mind can be summarized into the following three points in different stages of the VLBI procedures. The first step is in making an observation schedule for a VLBI experiment. We have to schedule as many low elevation observations as possible to keep the elongation of the error ellipsoid minimum. The second step is the parameter estimation. We can considerably reduce the amount of post-fit delay residuals by estimating the temporal variation of the atmosphere as a line with a lot of breaks. We can prevent the instability by introducing constraints for the continuity of the variation. As the last step, the selection of the atmospheric mapping function is important to avoid systematic errors of the estimated vertical positions.

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