Brueshaber et al. (2019) の方程式系

ここでは，Brueshaber et al. (2019) で用いられたモデルの定式化を行う．

1　方程式系

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla_z)\boldsymbol{u} = -f\boldsymbol{k}\times\boldsymbol{u} - g'\nabla_z h - \boldsymbol{D}_u, \tag{1.1}$$

$$\frac{\partial g'h}{\partial t} + g'\nabla\cdot(h\boldsymbol{u}) = \Sigma S_{storm} + S_{mass} + S_{rad} - D_h. \tag{1.2}$$

ここで，D は数値粘性項，h は上層の厚さ，g' は低減重力加速度，S_{storm} はストームを模した質量強制項，S_{mass} は質量調整項，S_{rad} は放射緩和項である[1]．それぞれの強制項は

$$S_{storm} = s\cdot\exp\left[-\frac{R^2}{R_{storm}^2} - \frac{(t-t_0)}{\tau_{storm}^2}\right], \tag{1.3}$$

$$S_{mass} = -\langle\Sigma S_{storm}\rangle, \tag{1.4}$$

$$S_{rad} = -\frac{\langle g'h\rangle - g'h_{eq}}{\tau_{mass}} - \frac{g'h - \langle g'h\rangle}{\tau_{APE}} \tag{1.5}$$

である．

[1]連続の式は Brueshaber et al. (2019) の記載が誤植と考え，Showman (2007) の記載を用いる．

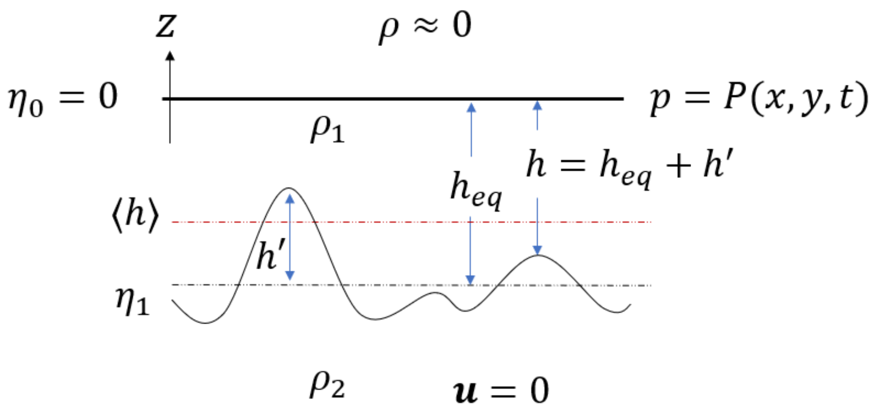

図 1: 1.5 層モデルの模式図

2 渦度発散型

ここでは，上記の方程式の球面渦度発散型を求める．1 層浅水モデルの方程式[2]を参考に渦度発散型に書きかえると，

$$\begin{aligned}\frac{\partial \zeta}{\partial t} &= -\frac{1}{a\cos\vartheta}\frac{\partial}{\partial \lambda}[(\zeta + f)u] - \frac{1}{a\cos\vartheta}\frac{\partial}{\partial\vartheta}[(\zeta+f)v\cos\vartheta] \\ &\quad - K_m\left[(-1)^{N_m}\nabla^{2N_m} - \left(\frac{2}{a^2}\right)^{2N_m}\right]\zeta, \end{aligned} \tag{2.6}$$

$$\begin{aligned}\frac{\partial D}{\partial t} &= \frac{1}{a\cos\vartheta}\frac{\partial}{\partial \lambda}[(\zeta + f)v] - \frac{1}{a\cos\vartheta}\frac{\partial}{\partial\vartheta}[(\zeta+f)u\cos\vartheta] \\ &\quad - \nabla^2[g'h + E] - K_m\left[(-1)^{N_m}\nabla^{2N_m} - \left(\frac{2}{a^2}\right)^{2N_m}\right]D, \end{aligned} \tag{2.7}$$

$$\begin{aligned}\frac{\partial h}{\partial t} &= -\frac{1}{a\cos\vartheta}\frac{\partial}{\partial \lambda}(hu) - \frac{1}{a\cos\vartheta}\frac{\partial}{\partial\vartheta}(hv\cos\vartheta) \\ &\quad + (\Sigma S_{strom} + S_{mass} + S_{rad})/g' - (-1)^{N_h}K_h\nabla^{2N_h}h. \end{aligned} \tag{2.8}$$

[2] http://www.ep.sci.hokudai.ac.jp/ rsuzuki/research/williamson/tex/shallow_eq_onelayer.pdf

ここで，$E=(u^2+v^2)/2$，K_m は超粘性係数，K_h は超拡散係数である．
$U=u\cos\vartheta, V=v\cos\vartheta$ と $\mu=\sin\vartheta$ を用い，サイン緯度に書きかえると

$$\begin{aligned}\frac{\partial\zeta}{\partial t} &= -\frac{1}{a(1-\mu^2)}\frac{\partial}{\partial\lambda}[(\zeta+f)U]-\frac{1}{a}\frac{\partial}{\partial\mu}[(\zeta+f)V] \\ &\quad -K_m\left[(-1)^{N_m}\nabla^{2N_m}-\left(\frac{2}{a^2}\right)^{2N_m}\right]\zeta, \qquad (2.9)\\ \frac{\partial D}{\partial t} &= \frac{1}{a(1-\mu^2)}\frac{\partial}{\partial\lambda}[(\zeta+f)V]-\frac{1}{a}\frac{\partial}{\partial\mu}[(\zeta+f)U] \\ &\quad -\nabla^2[g'h+E]-K_m\left[(-1)^{N_m}\nabla^{2N_m}-\left(\frac{2}{a^2}\right)^{2N_m}\right]D, \qquad (2.10)\\ \frac{\partial h}{\partial t} &= -\frac{1}{a(1-\mu^2)}\frac{\partial}{\partial\lambda}(hU)-\frac{1}{a}\frac{\partial}{\partial\mu}(hV) \\ &\quad +(\Sigma S_{strom}+S_{mass}+S_{rad})/g'-(-1)^{N_h}K_h\nabla^{2N_h}h. \qquad (2.11)\end{aligned}$$

また，上層の厚さ h を上層の厚さの平均場 h_{eq} とそこからのずれ h' を用いて，$h=h'(\lambda,\phi,t)+h_{eq}$ と書くと

$$\begin{aligned}\frac{\partial\zeta}{\partial t} &= -\frac{1}{a(1-\mu^2)}\frac{\partial}{\partial\lambda}[(\zeta+f)U]-\frac{1}{a}\frac{\partial}{\partial\mu}[(\zeta+f)V] \\ &\quad -K_m\left[(-1)^{N_m}\nabla^{2N_m}-\left(\frac{2}{a^2}\right)^{2N_m}\right]\zeta, \qquad (2.12)\\ \frac{\partial D}{\partial t} &= \frac{1}{a(1-\mu^2)}\frac{\partial}{\partial\lambda}[(\zeta+f)V]-\frac{1}{a}\frac{\partial}{\partial\mu}[(\zeta+f)U] \\ &\quad -\nabla^2[g'h'+E]-K_m\left[(-1)^{N_m}\nabla^{2N_m}-\left(\frac{2}{a^2}\right)^{2N_m}\right]D, \qquad (2.13)\\ \frac{\partial h'}{\partial t} &= -\frac{1}{a(1-\mu^2)}\frac{\partial}{\partial\lambda}(h'U)-\frac{1}{a}\frac{\partial}{\partial\mu}(h'V)-h_{eq}D \\ &\quad +(\Sigma S_{strom}+S_{mass}+S_{rad})/g'-(-1)^{N_h}K_h\nabla^{2N_h}h. \qquad (2.14)\end{aligned}$$

ここで，

$$\begin{aligned}S_{storm} &= s\cdot\exp\left[-\frac{R^2}{R^2_{storm}}-\frac{(t-t_0)}{\tau^2_{storm}}\right], \qquad (2.15)\\ S_{mass} &= -\langle\Sigma S_{storm}\rangle, \qquad (2.16)\\ S_{rad} &= -\frac{\langle g'h'\rangle}{\tau_{mass}}-\frac{g'h'-\langle g'h'\rangle}{\tau_{APE}} \qquad (2.17)\end{aligned}$$

である[3].

[3] S_{rad} の変形は

$$
\begin{aligned}
S_{rad} &= -\frac{\langle g'h\rangle - g'h_{eq}}{\tau_{mass}} - \frac{g'h - \langle g'h\rangle}{\tau_{APE}} \\
&= -\frac{\langle g'(h'+h_{eq})\rangle - g'h_{eq}}{\tau_{mass}} - \frac{g'(h'+h_{eq}) - \langle g'(h'+h_{eq})\rangle}{\tau_{APE}} \\
&= -\frac{\langle g'h'\rangle + g'h_{eq} - g'h_{eq}}{\tau_{mass}} - \frac{g'h' + g'h_{eq} - \langle g'h'\rangle - g'h_{eq}}{\tau_{APE}} \\
&= -\frac{\langle g'h'\rangle}{\tau_{mass}} - \frac{g'h' - \langle g'h'\rangle}{\tau_{APE}}.
\end{aligned}
$$